Use of a Variant to Measure New Events Converging

fixed

variables: a, b, c

invariants:
- inv1_1 : $a \in \mathbb{N}$
- inv1_2 : $b \in \mathbb{N}$
- inv1_3 : $c \in \mathbb{N}$
- inv1_4 : $a + b + c = n$
- inv1_5 : $a = 0 \lor c = 0$

ML_out
when
$a + b < d$
$c = 0$
then
$a := a + 1$
end

ML_in
when
$c > 0$
then
$c := c - 1$
end

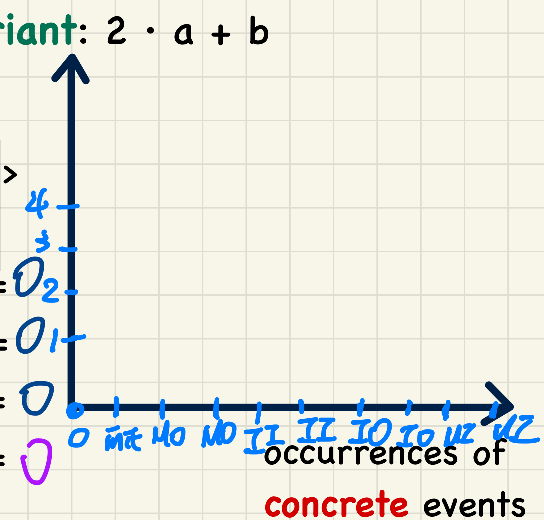
IL_in
when
$a > 0$
then
$a := a - 1$
$b := b + 1$
end

IL_out
when
$b > 0$
$a = 0$
then
$b := b - 1$
$c := c + 1$
end

Variants for New Events: $2 \cdot a + b$

<init, ML_out, ML_out, IL_in, IL_in, IL_out, IL_out, ML_in, ML_in>

	init	ML_out	ML_out	IL_in	IL_in	IL_out	IL_out	ML_in	ML_in
a	0	1	2	1	0	0	0	0	0
b	0	0	0	1	2	1	0	0	0
c	0	0	0	0	0	1	2	1	0
v	0	2	4	3	2	1	0	0	0



PO of Convergence/Non-Divergence/Livelock Freedom

Variants for New Events: 2 · a + b

Variant Stays Non-Negative

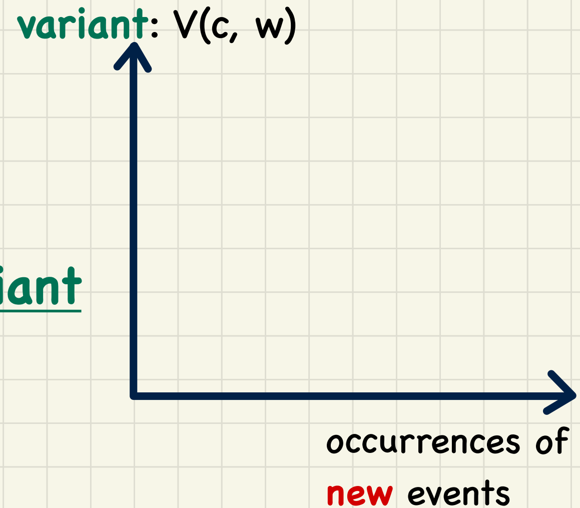
IL_in/NAT

$$\frac{\begin{array}{l} A(c) \\ I(c, v) \\ J(c, v, w) \\ H(c, w) \\ \vdash \\ V(c, w) \in \mathbb{N} \end{array}}{}\ \text{NAT}$$

A New Event Occurrence Decreases Variant

IL_in/VAR

$$\begin{array}{l} A(c) \\ I(c, v) \\ J(c, v, w) \\ H(c, w) \\ \vdash \\ V(c, F(c, w)) < V(c, w) \end{array}\quad \text{VAR}$$



Example Inference Rules

$$\frac{H, \neg P \vdash Q}{H \vdash P \vee Q} \textbf{ OR\_R}$$

$$\frac{H, P, Q \vdash R}{H, P \wedge Q \vdash R} \textbf{ AND\_L}$$

$$\frac{H \vdash P \qquad H \vdash Q}{H \vdash P \wedge Q} \textbf{ AND\_R}$$